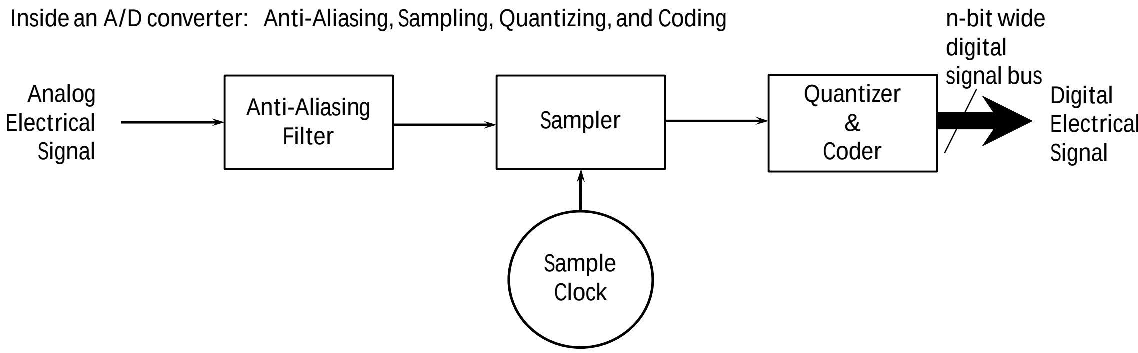
Analog-to-Digital Conversion

Inside an A/D converter: Anti-Aliasing, Sampling, Quantizing, and Coding



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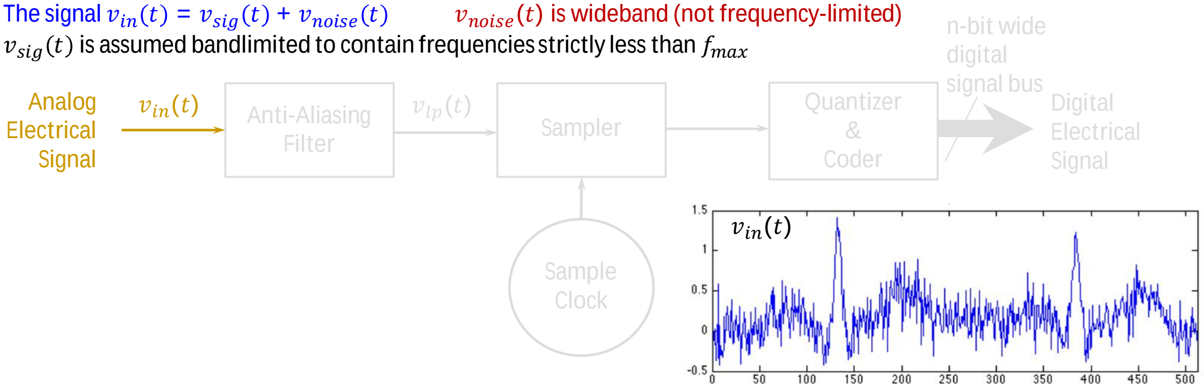
Analog-to-Digital Conversion

The signal $v_{in}(t) = v_{sig}(t) + v_{noise}(t)$ $v_{noise}(t)$ is wideband (not frequency-limited)

$v_{sig}(t)$ is assumed bandlimited to contain frequencies strictly less than f_{max}



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Analog-to-Digital Conversion

f_{max} = maximum valid signal. Anything above this is unwanted noise although it may exist.
$f_{nyquist}$ = cutoff freq. of filter. Anything above this should be attenuated.

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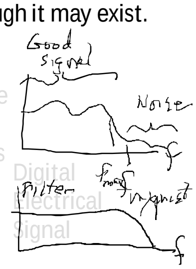
Analog Electrical Signal → $v_{in}(t)$ → Anti-Aliasing Filter → $v_{lp}(t)$ → Sampler → Quantizer & Coder → n-bit wide digital signal bus → Digital Electrical Signal

Sample Clock → Sampler

f_{max} ←
$f_{nyquist}$, above $f_{nyquist}$



The analog anti-aliasing filter serves to smooth the waveform out and make it continuous with respect to the sampling rate.

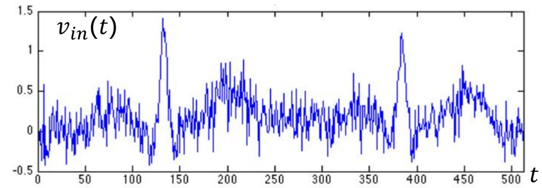
It should pass all the signal.

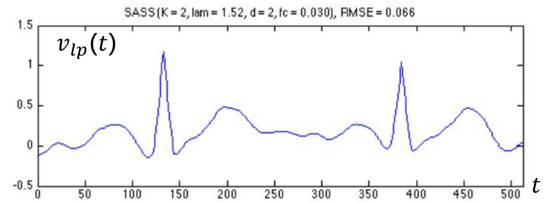
It should reject all frequencies above the signal frequencies.

Good-quality input amplifiers include the anti-aliasing filter. It can usually be configured for one of several choices for $f_{nyquist}$

Essential: $f_{nyquist} > f_{max}$

Typically: $f_{nyquist} > 1.1 f_{max}$ or even $f_{nyquist} > 10 f_{max}$





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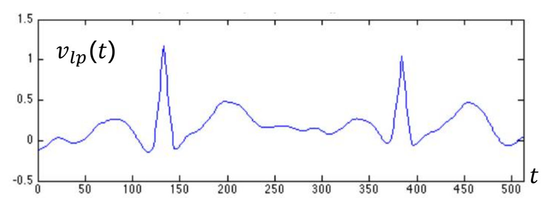
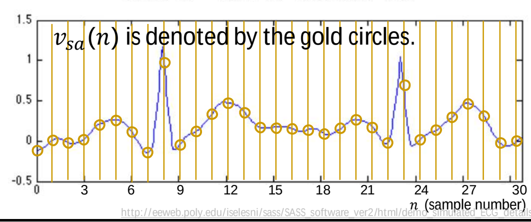
Frequency of sampling is f_s

The sampling system makes the signal discrete in terms of the independent variable (usually time or distance, etc.)

At each sample-clock time the sampler captures the voltage (or current) of the signal. The sample-value exactly matches the signal-value at the moment of sampling.

The sampled signal $v_{sa}(n)$ is continuous in voltage but discrete in time. $t = n/f_s$





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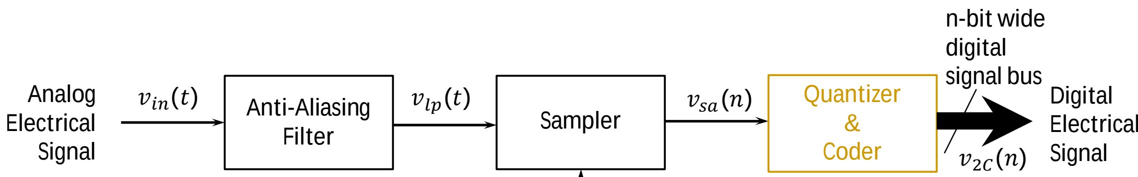
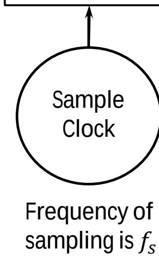
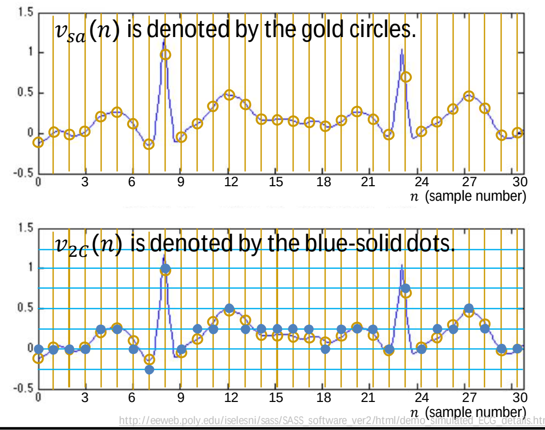


The quantizer and coder converts the analog sample-ordinates to a digital code, typically using the twos-complement code, which will be represented as one of the number formats available in your software system, e.g. "`int`" or "`long int`" etc.





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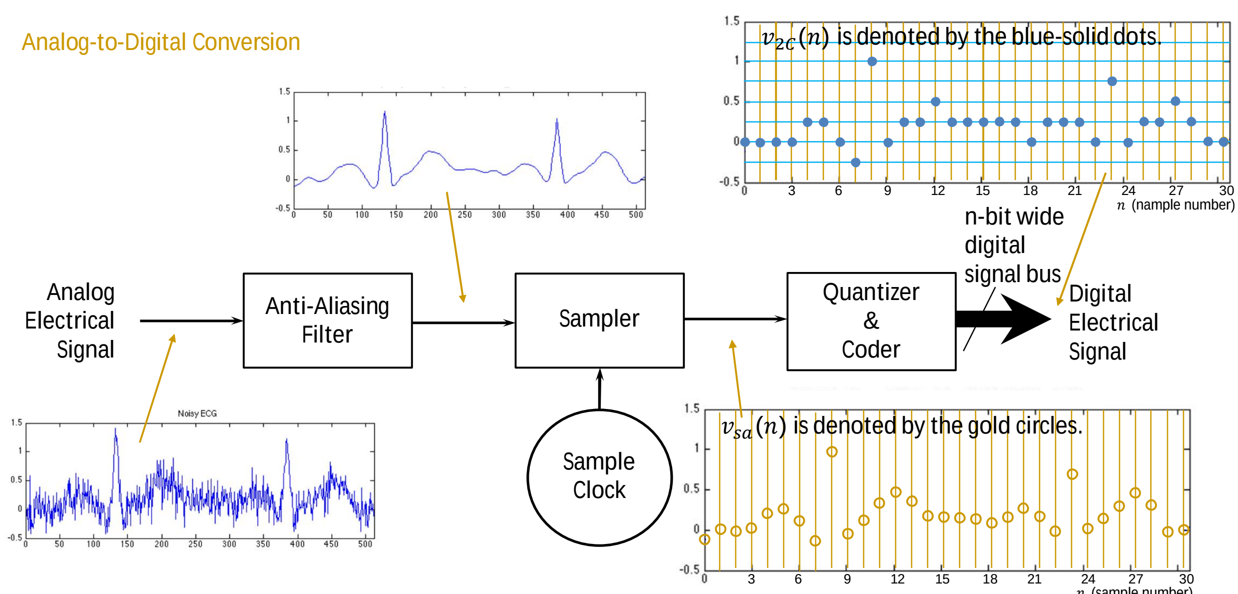

Ordinate (vertical-axis) values exactly match signal.
Ordinate values only defined for integer-valued abscissa (horizontal-axis) values.
The signal is absent between sample numbers.

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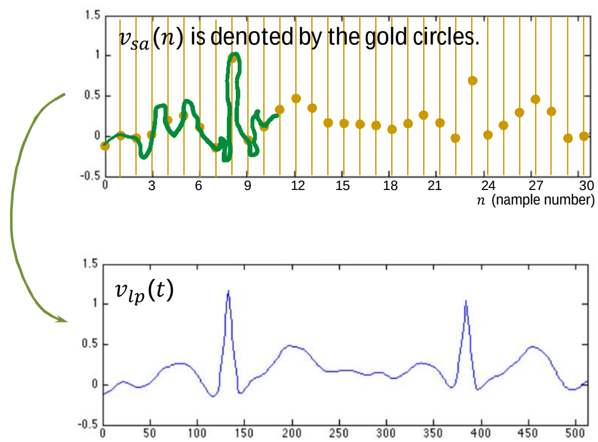
Analog-to-Digital Conversion

<u>The Nyquist-Shannon low-pass sampling theorem</u>:

If a discrete-time analog signal, $v_{sa}(n)$ was created by sampling an analog signal bandlimited to $f_{nyquist}$, and if the sampling frequency, was at least two times greater than $f_{nyquist}$,

$$f_s > 2f_{nyquist}$$

then $v_{tp}(t)$ can be perfectly reconstructed from $v_{sa}(n)$



<u>Corollary</u>: There is only one unique signal that will pass through all the samples of $v_{sa}(n)$ when the samples are spaced $1/f_s$ apart. The waveform between samples Is not arbitrary.

The drawn green line could not possibly have been an output of the anti-aliasing filter. The sharp bends in it have frequency-content above $f_{nyquist}$.

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Analog-to-Digital Conversion

The fly in the ointment:

Perfectly storing $f_{sa}(n)$ would require an infinite amount of memory
because each sample is real-valued. (Has an infinite number of decimal places.)

The quantizer reduces the amount of memory needed to a finite amount of storage.
It does this by rounding the sample-values to the nearest available quanta on the ordinate.
The quantizer adds quantization noise.

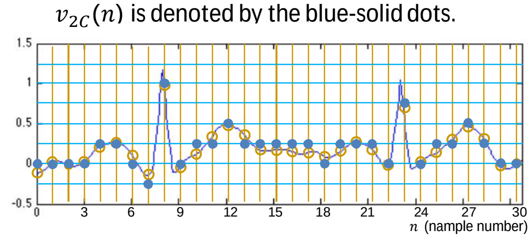
$$v_{2C} = v_{sa}(t) + v_{qn}(t)$$

Where $v_{qn}(t)$ is the quantization noise.

The signal-to-quantization-noise ratio is. . .
defined as a power ratio. . .
typically expressed in the units of decibels (dB)

$$SQNR_{dB} = 10\log_{10}\left(\frac{P_{2C}}{P_{qn}}\right)$$

$v_{sa}(n)$ is denoted by the gold circles.
$v_{2C}(n)$ is denoted by the blue-solid dots.



Where P_{2C} is the average power of the quantized signal and P_{qn} is the average power of the noise.

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Analog-to-Digital Conversion

Aliasing must be avoided.
It can only be avoided in the analog world!

The relationship between the time-domain and the sample-domain.

Suppose we have a sinusoidal analog signal, $v_{in}(t) = A_{peak}\cos(2\pi f_0 t)$

We wish to convert this to a digital signal.
The anti-aliasing filter passes this signal with no change because $f_0 < f_{nyquist}$.

$$v_{lp}(t) = v_{in}(t) = A_{peak}\cos(2\pi f_0 t)$$

The sampler evaluates this function every $1/f_s$ seconds. This means that at the sample times $t = n/f_s$

$$v_{sa}(n) = v_{lp}(n/f_s) = A_{peak}\cos((2\pi n f_0)/f_s)$$

Define F_o as the digital frequency of the signal. (And we call f_0 the analog frequency of the signal.)

$$F_0 \triangleq f_0/f_s$$

$$v_{sa}(t) = A_{peak}\cos(2\pi F_o n)$$

To satisfy the Nyquist-Shannon low-pass sampling theorem $f_o < f_{nyquist}$ and $f_{nyquist} < f_s/2$ thus $f_0 < f_s/2$

$$F_0 \triangleq \frac{f_0}{f_s} < \frac{f_s/2}{f_s} = \frac{1}{2}$$

Units: f_0 is in Hz (cycles/sec)
F_0 is in cycles/sample (both dim. Less)
f_s is in Hz

$$\boxed{F_0 < \frac{1}{2}}$$

Is your digital frequency greater than one-half? Oops!

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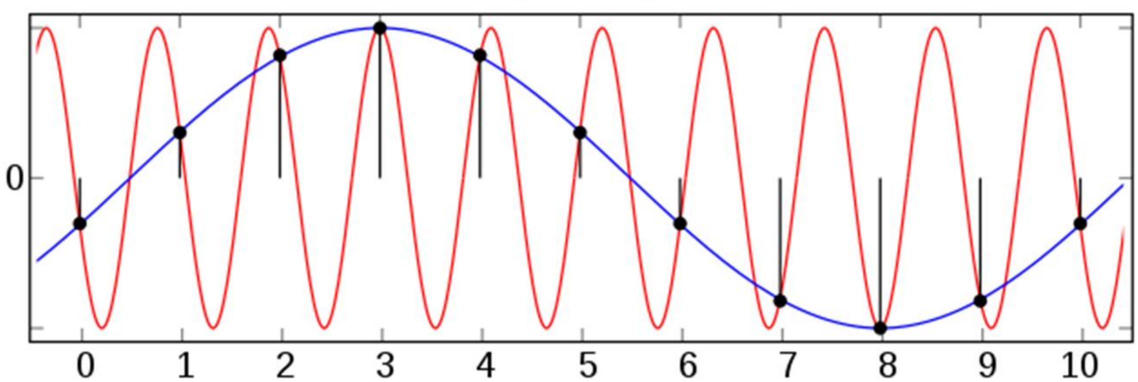

Demos: https://www.youtube.com/watch?v=UaKho805vCE
https://www.youtube.com/watch?v=XoVhNhi76Qk

Is your digital frequency greater than one-half? Oops!

https://commons.wikimedia.org/wiki/File:AliasingSines.svg

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